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New class of polyphase sequences with two-valued auto- and crosscorrelation functions

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Indexing term: Binary sequences

A new class of polyphase sequences with two-valued periodic auto- and crosscorrelation properties is proposed. It is proved that, for a given prime length $L > 3$, the out-of-phase ACFs and CCFs of the sequences are constant and equal to $\sqrt{L}$. It is also shown that sequences of the same length are mutually orthogonal and the correlation values asymptotically reach the Sarwate bound.

Introduction: Sets of sequences with good correlation properties have found application in radar, ranging and tracking, spread-spectrum communications, multiple-access communications, and system identification. For the correlation method of system identification, it is especially desirable that the out-of-phase autocorrelation functions (ACFs) of a sequence be constant. For multiple-access communication purposes it is also desirable that the pairwise crosscorrelation functions (CCFs) between sequences assigned to various users be small, and preferably constant. There are many well-known classes of sequences having two-valued ACFs and small CCFs. For example, maximal length sequences, quadratic residue sequences, Frank sequences, Chu sequences, Lüke sequences, etc. [1–4]. In this Letter, a new class of polyphase sequences with near-optimal two-valued ACFs and CCFs is proposed. It is proved that, for a given prime length $L > 3$, the out-of-phase ACFs and CCFs of the sequences are constant and equal to $\sqrt{L}$. An interesting and useful property of these sequences is that all the sequences of the same length L are mutually orthogonal. It is shown that the correlation values asymptotically reach the Sarwate bound.

Sequences with two-valued ACFs and CCFs: For any integers r, n and prime $L > 3$, where $0 \le n, r < L$, the new class of polyphase sequences, $a^{(r)} = (a_0^{(r)}, a_1^{(r)}, \ldots, a_{L-1}^{(r)})$, is defined as

$$a_n^{(r)} = \alpha^{n(n+1)(n+2)/6+rn} \qquad \alpha = e^{i2\pi v/L} \tag{1}$$

where α is a primitive Lth root of unity and v is any integer relatively prime to L. Obviously there exist L sequences each of length L for any prime $L > 3$.

We now show that the above sequences have the following ACF/CCF properties:

$$|R_{r,r}(\tau)| = \begin{cases} L & \tau = 0 \\ \sqrt{L} & \tau \neq 0 \end{cases} \tag{2}$$

$$|R_{r,s}(\tau)| = \begin{cases} 0 & \tau = 0,\ r \neq s \\ \sqrt{L} & \tau \neq 0 \end{cases} \tag{3}$$

The squared absolute value of the periodic CCF $R_{r,s}(\tau)$ between sequence $a^{(r)}$ and sequence $a(s)$ is defined as

$$|R_{r,s}(\tau)|^2 = \sum_{n=0}^{L-1} a_n^{(r)} a_{n+\tau}^{(s)*} \sum_{m=0}^{L-1} a_m^{(r)*} a_{m+\tau}^{(s)} \tag{4}$$

Substituting eqn. 1 into eqn. 4, we obtain

$$\begin{aligned} &|R_{r,s}(\tau)|^2 \\ &= \sum_{n=0}^{L-1} \alpha^{n(n+1)(n+2)/6+rn-(n+\tau)(n+\tau+1)(n+\tau+2)/6-s(n+\tau)} \\ &\sum_{m=0}^{L-1} \alpha^{-m(m+1)(m+2)/6-rm+(m+\tau)(m+\tau+1)(m+\tau+2)/6+s(m+\tau)} \\ &= \sum_{n=0}^{L-1} \alpha^{-\tau(2+6n+3n^2+3\tau+3n\tau+\tau^2)/6-(s-r)n-s\tau} \\ &\sum_{m=0}^{L-1} \alpha^{\tau(2+6m+3m^2+3\tau+3m\tau+\tau^2)/6-(s-r)m-s\tau} \\ &= \sum_{n=0}^{L-1}\sum_{m=0}^{L-1} \alpha^{\tau[6(m-n)+3(m^2-n^2)+3\tau(m-n)]/6+(s-r)(m-n)} \\ &= \sum_{n=0}^{L-1}\sum_{m=0}^{L-1} \alpha^{(m-n)[\tau(\tau+m+n+2)/2+(s-r)]} \end{aligned} \tag{5}$$

When $\tau = 0$, it is obvious that

$$|R_{r,s}(\tau)|^2 = \sum_{n=0}^{L-1}\sum_{m=0}^{L-1} \alpha^{(m-n)(s-r)} = \begin{cases} L^2 & \tau = 0,\ s = r \\ 0 & \tau = 0,\ s \neq r \end{cases} \tag{6}$$

When $\tau \neq 0$, we introduce the following change of variables:

$$n = m + l \qquad l = 0, 1, \ldots, L-1 \tag{7}$$

$|R_{r,s}(\tau)|^2$ can then be rewritten as

$$\begin{aligned} |R_{r,s}(\tau)|^2 &= \sum_{l=0}^{L-1}\sum_{m=0}^{L-1} \alpha^{-l[\tau(\tau+2m+l+2)/2+(s-r)]} \\ &= \sum_{l=0}^{L-1} \alpha^{-\tau l(\tau+l+2)/2-l(s-r)} \sum_{m=0}^{L-1} \alpha^{-\tau lm} \\ &= L \end{aligned} \tag{8}$$

where

$$\sum_{m=0}^{L-1} \alpha^{-\tau lm} = \begin{cases} 0 & l \neq 0,\ (\tau, L) = 1 \\ L & l = 0 \end{cases} \tag{9}$$

It should be noted that for $\tau = 0, 1, \ldots, L-1$, L must be a prime number in order to satisfy $(\tau, L) = 1$.

We now consider the asymptotic performance of the sequences. Sarwate [5] has shown that for a family of M uniform sequences, each of period L, the maximal magnitudes of the sidelobes Θ_a, Θ_c of auto- and crosscorrelation are lower bounded by

$$\frac{\Theta_c^2}{L} + \frac{L-1}{L(M-1)}\frac{\Theta_a^2}{L} \geq 1 \tag{10}$$

For the proposed sequences, $M = L$, $\Theta_a = \Theta_c = \sqrt{L}$, the Sarwate-bound yields

$$\frac{(\sqrt{L})^2}{L} + \frac{L-1}{L(L-1)}\frac{(\sqrt{L})^2}{L} = 1 + \frac{1}{L} \geq 1 \tag{11}$$

which approaches the bound for large L. Therefore the above sequences are asymptotically optimal.

Example: As a simple example, let $L = 7$; we obtain seven distinct sequences:

$$\begin{aligned} a^{(0)} &= (1, & \alpha, & \alpha^4, & \alpha^3, & \alpha^6, & 1, & 1) \\ a^{(1)} &= (1, & \alpha^2, & \alpha^6, & \alpha^6, & \alpha^3, & \alpha^5, & \alpha^6) \\ a^{(2)} &= (1, & \alpha^3, & \alpha, & \alpha^2, & 1, & \alpha^3, & \alpha^5) \\ a^{(3)} &= (1, & \alpha^4, & \alpha^3, & \alpha^5, & \alpha^4, & \alpha, & \alpha^4) \\ a^{(4)} &= (1, & \alpha^5, & \alpha^5, & \alpha, & \alpha, & \alpha^6, & \alpha^3) \\ a^{(5)} &= (1, & \alpha^6, & 1, & \alpha^4, & \alpha^5, & \alpha^4, & \alpha^2) \\ a^{(6)} &= (1, & 1, & \alpha^2, & 1, & \alpha^2, & \alpha^2, & \alpha) \end{aligned}$$

where $\alpha = e^{i2\pi/7}$. Their auto- and crosscorrelation functions are given by

$$|R_{r,r}(\tau)| = (7,\ 2.6,\ 2.6,\ 2.6,\ 2.6,\ 2.6,\ 2.6)$$
$$|R_{r,s}(\tau)| = (0,\ 2.6,\ 2.6,\ 2.6,\ 2.6,\ 2.6,\ 2.6)$$

where $r \neq s$.

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New TCM codes for 4PSK-2PSK modulation

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Indexing terms: Phase shift keying, Trellis codes

The authors present some new multi-dimensional (3-D, 4PSK-2PSK) and 180° rotationally invariant trellis codes that combined with the demodulator (which locks onto the 2PSK signal of the 3-D signal set) allows robust operation at low signal to noise ratios. Examples of the codes are presented for 2, 4, 8, 16 and 32 states. The codes achieve a coding gain of 1.76dB (for two encoder states) to 5.44dB (for 32 encoder states) compared to uncoded BPSK. Distance profiles of the codes are shown.

New TCM codes for 4PSK-2PSK modulation: The authors present some new multi-dimensional (3-D, 4PSK-2PSK) and 180° rotationally invariant trellis codes that combined with the demodulator (which locks onto the 2PSK signal of the 3-D signal set) allows robust operation at low signal to noise ratios. Examples of the codes are presented for 2, 4, 8, 16 and 32 states. The codes achieve a coding gain of 1.76dB (for two encoder states) to 5.44dB (for 32 encoder states) compared to the uncoded BPSK. Distance profiles of the codes are shown.

Introduction: When QPSK signals are encoded with a linear, rate 1/2, convolutional code, it is only possible to achieve code invariance under 180° signal rotation with certain codes, whereas with other codes no such invariance exists [1]. The lack of invariance for 90° signal rotations requires that the QPSK signals be demodulated with the correct phase before Viterbi decoding. New non-linear 90° rotation invariant coded QPSK schemes were found in [1].

Another approach to the above problem is considered in this Letter. We consider linear TCM schemes with a 3-D constellation for sending one information bit per signalling interval. Each signal element is transmitted in the two consecutive signalling intervals as a pair of 2PSK and 4PSK symbols (Fig. 1*a*). This technique has been used previously with 4PSK-8PSK [2].

For coherent detection we have to extract the carrier phase from the received signal. In our case, for 2PSK-4PSK modulation we have the option of performing synchronisation with only two phase states (on the 2PSK signal set). Thus only 180° synchronisation is required, increasing the robustness of the demodulator to noise. When 2PSK-4PSK signals are encoded, it is possible to easily achieve code invariance under 180° signal rotation. Such codes were found for 2, 4, 8, 16 and 32 encoder states.

3-D signal configuration and its partitioning: Each signal element in the 3-D signal space is created by 2PSK and 4PSK signals transmitted in two consecutive signalling intervals. Fig. 1*a* displays the

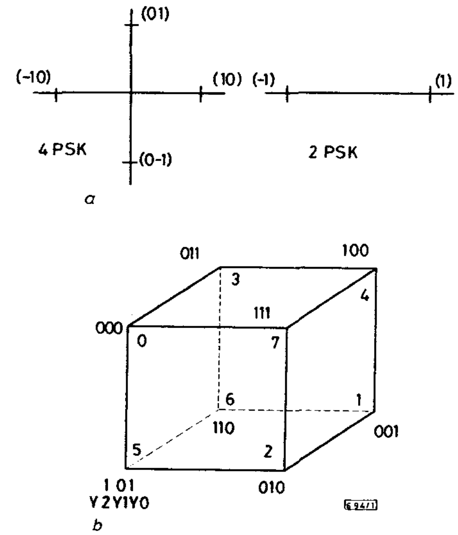

Fig. 1 *2PSK and 4PSK signal configuration, and 3-D signal space mapping for 180° rotationally invariant TCM codes*

a 2PSK and 4PSK signal configuration
b 3-D signal space mapping

2PSK and 4PSK signal configurations. The average signal energy is one. Each point in the 3-D signal configuration is created by a pair of 2PSK and 4PSK signals. We assign binary numbers $Y_2Y_1Y_0$ to the signal elements as shown in Fig. 1*b*. We notice that a 180° signal rotation always gives another signal with unchanged bits Y_2Y_1 and complementary bit Y_0. The minimum squared Euclidean distance within the 3-D signal set {0, 1, 2, 3, 4, 5, 6, 7} is 2, within the two subsets {0, 2, 4, 6} and {1, 3, 5, 7} the distance is 4, and within subsets {0, 4}, {2, 6}, {1, 5}, {3, 7} the distance is 6.

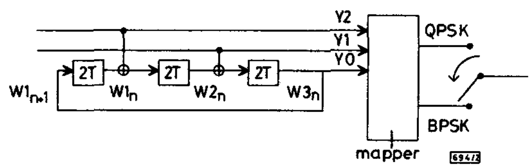

Fig. 2 *Eight state encoder*

Code design example: The design procedure follows the steps described in [3]. After set partitioning and state transition diagram definition the signal elements are assigned to the encoder state transitions. The assignment is performed such that the three Ungerboeck rules [4] are applied and rule A1 [3] is satisfied. To describe the code we use the parity check equation as given in [4]. The encoder in Fig. 2 has (in octal notation) $h^2 = 04$, $h^1 = 02$ and $h^0 = 11$. Because $H^0(D) = D^3 \oplus 1$ and $H^0(1) = 0$, the code is 180° invariant [7]. Distance d^2_{min} for the code of Fig. 2 is 10, hence the asymptotic coding gain of the code compared to uncoded BPSK is 10log(10/4) = 3.98dB. Because Y_2Y_1 do not change on phase rotation, a differential encoder is not necessary.

Table 1: Code examples

υ	k	h^2	h^1	h^0	d^2_{free}	N_{free}	γ
							dB
1	1	–	2	3	6	2	1.76
2	2	4	6	3	8	4	3.01
3	2	04	02	11	10	1	3.98
4	2	14	02	33	12	2	4.77
5	2	04	12	71	14	3	5.44

(Notation as in [7])